A priori mixed baryons and weak radiative decays

A. García*
Departamento de Física.
Centro de Investigación y de Estudios Avanzados del IPN.
A.P. 14-740, C.P. 07000, México, D.F., MEXICO.

R. Huerta† and G. Sánchez-Colón‡
Departamento de Física Aplicada.
Centro de Investigación y de Estudios Avanzados del IPN.
Unidad Mérida.
A.P. 73, Cordemex 97310, Mérida, Yucatán, MEXICO.

(February 6, 1996)

A priori mixings of eigenstates in physical states are quantum mechanical effects well known in several realms of physics. The possibility that such effects are also present in particle physics, in the form of flavor and parity mixings, is studied. An application to weak radiative decays of hyperons is discussed. It is suggested that this scheme may also be present in non-leptonic and rare mode decays as the enhancement phenomenon.

PACS Numbers: 13.40.Hg, 11.30.Er, 11.30.Hr, 12.60.-i

I. INTRODUCTION

Because parity and strong flavors (strangeness, charm, etc.) are violated in nature, the physical (mass eigenstates) hadrons cannot be either parity or flavor eigenstates, i.e., the former must be admixtures of the latter. It is generally believed that the breaking of flavor global groups is caused by the mass differences of hadrons, but in such a way that parity and all flavors are conserved, i.e., the mass operator of hadrons giving rise to such breakings does not contain a piece that violates parity and flavor. The flavor and parity mixings in physical hadrons are attributed to the perturbative intervention of $W^{\pm}_{\mu}$ and Z^0_{μ} (parity mixing only). And, precisely because such intervention is perturbative, such mixings can appear only in higher orders of perturbation theory; thus, such mixings appear, so to speak, *a posteriori.*

However, the possibility that the mass operator of hadrons does contain a (necessarily) very small piece that is flavor and parity violating is not excluded by any fundamental principle. If such a piece does exist, then, the parity and flavor admixtures in hadrons must come *a priori*, in a non-perturbative way. It is not idle to emphasize that such a piece could not be attributed to the $W^{\pm}_{\mu}$ and Z^0_{μ}.

Our purposes in this paper are (i) to explore the possibility that *the mass operator of hadrons contain flavor and parity violating pieces leading to a priori mixings,* (ii) to study how to implement the a priori mixings in hadrons, and (iii) to illustrate the potential usefulness such mixings might have. Accordingly, in Sec. II we discuss how a priori mixings may be introduced at the hadron level via an ansatz, and in Sec. III we apply a priori mixings to weak radiative decays of hyperons in order to show how the framework we introduced can be used. We reserve the last section to discuss the potential implications of a priori mixings in particle physics.

To close this section, let us remark that a priori mixings are quantum mechanical effects well known in other realms of physics, e.g., atomic physics. Thus, another way to put the aims of this paper is to explore the questions whether a priori mixings are also present in particle physics and what consequences this could have.

II. AN ANSATZ

The implementation of a priori mixings for practical applications cannot, as of today, be achieved from first principles, i.e., by starting from a model at the quark level and then performing the QCD calculations to obtain the physical hadrons and their couplings. In order to proceed we must elaborate an ansatz. We shall do this in a series of steps (or working hypothesis) and we shall restrict what follows to spin 1/2 baryons.

Our ansatz consists of the following steps:

S1. In addition to ordinary or s-baryons there exist p-baryons. Let us assume that the s-baryons have intrinsic parity opposite to the one of the p-baryons. This is a crucial assumption in our approach. The indeces s and p refer

to this, s means positive intrinsic parity and p means negative intrinsic parity. Both sets have the same strong-flavor assignment and belong to two different 20 representations of SU_4.

S2. There exist very small flavor and parity violating pieces in the mass operator for such baryons and the passage to the physical baryons is performed by a final rotation $R = (r_{ij})$ that diagonalizes the mass operator. R will be considered real for simplicity and since we are not taking into account the CP-violation problem in baryon decays. This leads to a priori flavor and parity admixtures in the physical (mass eigenstates) baryons, for example, like $\Lambda_{ph} = \Lambda_s + \alpha n_s + \alpha' n_p + \beta \Xi^0_s + \beta' \Xi^0_p + \cdots$. We do not know how to fix the matrix elements of R, but on experimental grounds we can advance that the mixing angles are very small, so that, $r_{ij} = \delta_{ij} + \epsilon_{ij}$, with $\epsilon_{ji} = -\epsilon_{ij}$ and $i, j = 1, \ldots, 40$.

S3. The small mixing parameters (α, α', β, etc.) are determined by assigning strong-flavor group properties to the transformation matrix R. For example, for SU_3 octets:

$$R = 1 + aU_+ + bU_- + cO_+ + dO_- + a'\hat{U}_+ + b'\hat{U}_- + c'\hat{O}_+ + d'\hat{O}_- + \cdots, \tag{1}$$

where $U_\pm$, $\hat{U}_\pm$, $O_\pm$, and $\hat{O}_\pm$, are all U-spin (charge conserving) ladder operators, with $U_\pm$ and $O_\pm$ ($\hat{U}_\pm$ and $\hat{O}_\pm$) acting on s-baryons (p-baryons). The $U_\pm$ and $\hat{U}_\pm$ operators connect hadrons in the same representation, so that, they are generators, but $O_\pm$ and $\hat{O}_\pm$ are not, of necessity, because they can connect hadrons that belong to different representations. With the property $RR^\dagger = R^\dagger R = I$ and if we choose the symmetric D-type couplings of $O_\pm$ and $\hat{O}_\pm$ equal to zero, then the a priori flavor and parity mixings for SU_3 octets can be described in terms of only four independent mixing angles named: σ, δ, δ', and $\hat{\sigma}$. We must point out that the previous rules in this step have a parallelism at the quark level so that they should be necessary to develop a formulation at that level. This matter will not be tried here.

Step S3 leads to [1]

$$p_{ph} = p_s + \sigma\Sigma^+_s + \delta\Sigma^+_p + \cdots$$

$$\Sigma^+_{ph} = \Sigma^+_s - \sigma p_s + \delta' p_p + \cdots$$

$$\Sigma^-_{ph} = \Sigma^-_s + \sigma\Xi^-_s + \delta\Xi^-_p + \cdots$$

$$\Xi^-_{ph} = \Xi^-_s - \sigma\Sigma^-_s + \delta'\Sigma^-_p + \cdots \tag{2}$$

$$n_{ph} = n_s + \sigma(\frac{1}{\sqrt{2}}\Sigma^0_s + \sqrt{\frac{3}{2}}\Lambda_s) + \delta(\frac{1}{\sqrt{2}}\Sigma^0_p + \sqrt{\frac{3}{2}}\Lambda_p) + \cdots$$

$$\Lambda_{ph} = \Lambda_s + \sigma\sqrt{\frac{3}{2}}(\Xi^0_s - n_s) + \delta\sqrt{\frac{3}{2}}\Xi^0_p + \delta'\sqrt{\frac{3}{2}}n_p + \cdots$$

$$\Sigma^0_{ph} = \Sigma^0_s + \sigma\frac{1}{\sqrt{2}}(\Xi^0_s - n_s) + \delta\frac{1}{\sqrt{2}}\Xi^0_p + \delta'\frac{1}{\sqrt{2}}n_p + \cdots$$

$$\Xi^0_{ph} = \Xi^0_s - \sigma(\frac{1}{\sqrt{2}}\Sigma^0_s + \sqrt{\frac{3}{2}}\Lambda_s) + \delta'(\frac{1}{\sqrt{2}}\Sigma^0_p + \sqrt{\frac{3}{2}}\Lambda_p) + \cdots$$

We have displayed only the predominantly ordinary matter physical baryons in terms of baryons that correspond to SU_3 octets, so that only three independent mixing angles σ, δ, and δ' survive in this calculation. The mixings with the other baryons corresponding to the 20 representations of SU_4 are similar to the above ones. In Eqs. (2) the dots stand for the latter flavor and parity mixings.

We have in mind an application to the observed weak radiative decays of hyperons. In this respect we introduce two more steps.

S4. The e.m. current operator J^{em}_μ for baryons is a flavor conserving Lorentz proper vector.

S5. The leading form factors f_1 in the matrix elements of J_μ^{em} between s and s, s and p, and p and p baryons are governed by the e.m. charge operator and the induced form factors f_2 are independent of the s and p indeces (because of hermiticity, the sign of f_2 in the matrix elements between p and s baryons must be reversed w.r.t. the sign of f_2 in the matrix elements between s and p baryons).

We wish to caution the reader that in assumption $S5$ the subindices s and p in the form factors f_2 should not be confused and taken to mean that they correspond to transition matrix elements between predominantly ordinary matter baryons and predominantly mirror matter baryons. This is important because the dimensionful magnetic-type f_2 depend on a mass scale determined by the masses of the physical baryons used. In Eqs. (2) the masses are of the order of 1 GeV and the pieces of the matrix elements of J_μ^{em} between these baryons that carry the indeces s and p have a mass scale of this 1 GeV order. If one were to compute transitions between a predominantly ordinary matter baryon and a predominantly mirror matter baryon then, of course, the mass scale would be dominated by the mass of the latter baryon, a scale which is unknown and by necessity must be very large. In the next section we shall be concerned with transitions between predominantly ordinary matter baryons exclusively.

III. AN APPLICATION

Our paper would not be complete if we did not attempt an application of the physical baryons with the non-perturbative a priori mixings of flavor and parity eigenstates. A most direct application we may have is the weak radiative decays of hyperons, although admittedly these may not necessarily be the easiest physical processes to understand.

The important point to remark is that, in contrast to $W_\mu^\pm$ mediated weak radiative decays, a priori mixed baryons can produce weak radiative decays via the ordinary electromagnetic interaction hamiltonian $H_{int}^{em} = eJ_\mu^{em}A^\mu$, where J_μ^{em} is the familiar e.m. current operator which is a flavor conserving Lorentz proper four-vector. That is, a priori mixings in baryons lead to weak radiative decays that in reality are ordinary parity and flavor conserving radiative decays, whose transition amplitudes are non-zero only because physical baryons are not flavor and parity eigenstates. Nevertheless, we use the standard notation "weak radiative decays" to bring the attention of the experts in this area.

The radiative decay amplitudes we want are given by the usual matrix elements $\langle\gamma, B_{ph}|H_{int}^{em}|A_{ph}\rangle$, where A_{ph} and B_{ph} stand for hyperons. A very simple calculation leads to the following hadronic matrix elements

$$\langle p_{ph}|J_{em}^\mu|\Sigma_{ph}^+\rangle = \bar{u}_p[\sigma(f_2^{\Sigma^+} - f_2^p) + (\delta' f_2^p - \delta f_2^{\Sigma^+})\gamma^5] i\sigma^{\mu\nu} q_\nu u_{\Sigma^+}$$

$$\langle \Sigma_{ph}^-|J_{em}^\mu|\Xi_{ph}^-\rangle = \bar{u}_{\Sigma^-}[\sigma(f_2^{\Xi^-} - f_2^{\Sigma^-}) + (\delta' f_2^{\Sigma^-} - \delta f_2^{\Xi^-})\gamma^5] i\sigma^{\mu\nu} q_\nu u_{\Xi^-}$$

$$\begin{aligned}\langle n_{ph}|J_{em}^\mu|\Lambda_{ph}\rangle = \bar{u}_n \Bigg\{ &\sigma \left[\sqrt{\frac{3}{2}}(f_2^\Lambda - f_2^n) + \frac{1}{\sqrt{2}} f_2^{\Sigma^0\Lambda}\right] \\ &+ \left[\sqrt{\frac{3}{2}}(\delta' f_2^n - \delta f_2^\Lambda) - \delta \frac{1}{\sqrt{2}} f_2^{\Sigma^0\Lambda}\right]\gamma^5 \Bigg\} i\sigma^{\mu\nu} q_\nu u_\Lambda \end{aligned} \tag{3}$$

$$\begin{aligned}\langle \Lambda_{ph}|J_{em}^\mu|\Xi_{ph}^0\rangle = \bar{u}_\Lambda \Bigg\{ &\sigma \left[\sqrt{\frac{3}{2}}(f_2^{\Xi^0} - f_2^\Lambda) - \frac{1}{\sqrt{2}} f_2^{\Sigma^0\Lambda}\right] \\ &+ \left[\sqrt{\frac{3}{2}}(\delta' f_2^\Lambda - \delta f_2^{\Xi^0}) + \delta' \frac{1}{\sqrt{2}} f_2^{\Sigma^0\Lambda}\right]\gamma^5 \Bigg\} i\sigma^{\mu\nu} q_\nu u_{\Xi^0} \end{aligned}$$

$$\begin{aligned}\langle \Sigma_{ph}^0|J_{em}^\mu|\Xi_{ph}^0\rangle = \bar{u}_{\Sigma^0} \Bigg\{ &\sigma \left[\frac{1}{\sqrt{2}}(f_2^{\Xi^0} - f_2^{\Sigma^0}) - \sqrt{\frac{3}{2}} f_2^{\Sigma^0\Lambda}\right] \\ &+ \left[\frac{1}{\sqrt{2}}(\delta' f_2^{\Sigma^0} - \delta f_2^{\Xi^0}) + \delta' \sqrt{\frac{3}{2}} f_2^{\Sigma^0\Lambda}\right]\gamma^5 \Bigg\} i\sigma^{\mu\nu} q_\nu u_{\Xi^0} \end{aligned}$$

The spinors u_A, $A = p, \Sigma^+$, etc. are ordinary four-component Dirac spinors and $q = p_B - p_A$. In accordance with S5, in Eqs. (3) we have used the generator properties of the electric charge, which require $f_{1s}^p = f_{1s}^{\Sigma^+} = 1$, etc. and

also, since s and p baryons belong to different irreducible representations, $f^{p}_{1sp} = f^{\Sigma^+}_{1sp} = 0$, etc. In addition, we have dropped the indices s and p in the f_2, so that $f^{p}_{2s} = f^{p}_{2sp} \neq f^{\Sigma^+}_{2s} = f^{\Sigma^+}_{2sp}$, etc. All the matrix elements are of the form $\bar{u}_B(C + D\gamma^5)i\sigma^{\mu\nu}q_\nu u_A$, where C and D would, respectively, correspond to the parity conserving and parity violating amplitudes of the $W^{\pm}_{\mu}$ mediated decays, although in our case both amplitudes are indeed parity conserving. Notice that Eqs. (3) comply with e.m. gauge invariance.

We shall compare Eqs. (3) with experiment, ignoring the contributions of $W^{\pm}_{\mu}$ amplitudes. We shall do this in order to be able to appreciate to what extent a priori mixings provide on their own right a framework to describe weak radiative decays.

To be able to proceed, we must decide what are the f_2 form factors in Eqs. (3). They are anomalous magnetic moment transition form factors, because, for example, $f^{\Sigma^+}_{2}$ corresponds to a form factor between Σ^+ flavor eigenstates present in the incoming physical Σ^+ with mass m_{Σ^+} and in the outgoing physical p with mass m_p. The f_2 form factors are affected by the masses of physical states. However, we shall assume that as a first approximation such mass dependence may be ignored. In this case, the f_2 in Eqs. (3) may be identified with the measured anomalous magnetic moments of the hyperons, i.e., $f^{A}_{2} = \mu^{exp}_{A} - e_A/e_p$ (in nuclear magnetons). Only $f^{\Sigma^0}_{2}$ is not measured [2], we shall use its SU_3 estimate, $\mu_{\Sigma^0} = (\mu_{\Sigma^+} + \mu_{\Sigma^-})/2$, as its central value with a 10% error bar. Also, we allow a 6% theoretical error in all the others.

The unknown quantities in Eqs. (3) are σ, δ, and δ'. We have no theoretical argument available to try to fix their values. We must leave them as free parameters and extract their values from experiment. For this purpose amplitudes (3) should be plugged into the usual formulas for the decay rates and angular asymmetries. These formulas and the experimental data can be found in Ref. [2]. The results are displayed in Table I. The values obtained for the a priori mixing angles are

$$\begin{aligned} \sigma &= (1.4 \pm 0.3) \times 10^{-6} \\ \delta &= (-0.35 \pm 0.13) \times 10^{-6} \\ \delta' &= (-0.22 \pm 0.13) \times 10^{-6} \end{aligned} \tag{4}$$

From Table I one can see that, given its simplicity, the above weakly mixed baryon scheme provides a qualitative reasonable description of weak radiative decays of hyperons. For completeness, our results may be compared with those obtained when the W-boson is responsible for these decays. This path has been extensively discussed, very recent reviews are found in Ref. [3]. All the models considered so far contain three or more free parameters, most of them are fixed with non-leptonic hyperon decays data. The main conclusion of Ref. [3] is that we still do not have a satisfactory theoretical explanation of weak radiative decays of hyperons. In this respect, it is important to remark that following our approach the calculations are appreciably simpler.

Nevertheless, it must be stressed that these results must be taken only as qualitative and not as quantitative. Given the simplicity of the above approach we find them encouraging enough as to take the a priori mixings in hadrons as a serious possibility.

IV. DISCUSSION AND CONCLUSIONS

In the previous sections we have explored the possibility that flavor and parity violating pieces in the mass operator of hadrons may exist. In this case, physical hadrons would show non-perturbative mixings of flavor and parity eigenstates, i.e., right from the start. These we have called a priori mixings to distinguish them from the mixings originated by the intervention of the $W^{\pm}_{\mu}$ and Z^{0}_{μ} bosons, which are perturbative and lead to such mixings in hadrons, but in an a posteriori fashion.

If a priori mixings are present, then weak decays may go via the flavor and parity conserving hamiltonians of strong and electromagnetic interactions. That is, with these mixings there would exist another mechanism to produce weak radiative, non-leptonic, and rare mode decays of hadrons, in addition to the already existing mechanisms provided by the $W^{\pm}_{\mu}$ and Z^{0}_{μ} bosons. One is immediately led to several questions: if a priori mixings in hadrons do exist in nature, how do their contributions compare to those of the $W^{\pm}_{\mu}$?, can their contributions be relevant?, and if so, would they improve our understanding of weak decays of hadrons?

Before discussing these questions one must first be able to calculate such contributions. This is not an easy task; however, one can introduce working hypotheses, based on educated guesses as much as possible. This we have done in Sec. II for spin 1/2 baryons. This collection of working hypotheses or ansatz enabled us to perform some calculations. As an illustration, we made an application to weak radiative decays of hyperons, in Sec. III. In order to keep things still at a relatively simple level, we introduced some approximations and, because of this, the results obtained should

be judged as qualitative only. We find them to be encouraging enough as to conclude that a priori mixings in hadrons should be taken seriously, as a novel possibility in Particle Physics.

Let us retake the above questions. As we mentioned in Sec. III, we lack any theoretical argument to roughly estimate the size of the a priori mixing angles. Clearly, it could well be the case that they are non-zero, so that this new effect does exist in Particle Physics as it does in other realms of physics, but they are extremely small. This would mean that with even very precise data a priori mixings would go undetected. In other words, the effect might exist but it would be a theoretical curiosity, irrelevant for practical purposes. The next possibility would be that the mixing angles be such that they lead to observable weak decays comparable to those mediated by $W^{\pm}_{\mu}$. In this case, one would have to face the complicated situation of disentangling what belongs to what in describing experimental data. The last possibility is that the a priori mixing angles be such that they lead to contributions appreaciably larger than the corresponding ones of $W^{\pm}_{\mu}$. In-as-much as a priori mixings are concerned, this is the really interesting situation. Their experimental predictions could then be subject to conclusive tests. Therefore, it is this last possibility we shall concentrate upon.

In the understanding of non-leptonic, weak radiative, and rare mode decays of hadrons a long-standing problem still remains an open challenge. This is the enhancement phenomenon. An impressive amount of effort has been invested in trying to demonstrate that the strong interactions that dress the hadron weak decays mediated by $W^{\pm}_{\mu}$ are responsible for such enhancement. The results so far are disappointing. It is commonly believed that the reason for this failure is our inability to compute with QCD, but once we can calculate better this problem will be solved favorably. Along this line of reasoning, the situation envisaged is that the intermediation of $W^{\pm}_{\mu}$ will saturate all measurements on flavor changing decays of hadrons and if any other mechanism exists it will necessarily be negligibly small, e.g., a priori mixings could not go beyond the theoretical curiosity level we just mentioned. However, it may happen that — once we can calculate better with QCD and contrary to expectations — it is demonstrated that enhancement cannot be produced by strong interactions. In this situation a new mechanism would be required.

This last comment provides the means to subject a priori mixings to critical tests. One of these is that, if they are to be an interesting effect in hadron weak decays, they should produce the observed enhancement phenomenon. Another very important one is that one should expect that the a priori mixing angles show a universality-like property, i.e., that their values appear resonably stable in different types of weak decays. However the judgement of how these tests and others are passed or failed will also be limited in the near future by our inability to calculate better with QCD. Accordingly, one should first expect to obtain relevant qualitative results and afterwards quantitative results based on educated guesses and simple models as we have illustrated in Secs. II and III. Clearly, it is along these lines that efforts of future research in this subject should be addressed. Also, the contributions of $W^{\pm}_{\mu}$ should be included at some point at a, for consistency, small level, say, by assuming that $|\Delta I| = 1/2$ amplitudes are of the same order of magnitude as the $|\Delta I| = 3/2$ amplitudes.

To close this paper and in the light of this discussion, we must stress that our application to weak radiative decays of hyperons should be taken more than anything else just as an exercise to learn to use a priori mixings of baryons. A more detailed analysis of these decays should be retaken later on. Nevertheless, for the time being we may point out that the lesson in Sec. III is encouraging enough so as to take with seriousness the possibility of the existence of this effect in Particle Physics.

ACKNOWLEDGMENTS

The authors wish to acknowledge useful discussions with J. L. Díaz Cruz, P. Kielanowski, G. Lopez Castro, J. L. Lucio, M. A. Pérez, J. Pestieau, M. H. Reno, and F. Yndurain. This work was partially supported by CONACYT (México).

* e-mail address (Internet): augarcia@fis.cinvestav.mx
† e-mail address (Internet): rhuerta@kin.cieamer.conacyt.mx
‡ e-mail address (Internet): gsanchez@mar.cieamer.conacyt.mx

[1] We follow the conventions of: D. B. Lichtenberg, *Unitary Simmetry and Elementary Particles* (Academic Press, Inc., N.Y., 1978), 2^{nd} Ed.
[2] Review of Particle Properties, Phys. Rev. **D 50**, 1173 (1994).
[3] B. Bassalleck, Nucl. Phys. A **547**, 299c (1992); J. Lach and P. Żenczykowski, Int. J. Mod. Phys. **A10**, 3817 (1995).

TABLE I. Predictions for the asymmetries and branching fractions (in units of 10^{-3}) of the weak radiative decays considered, along with the eight experimental measurements from Ref. [2].

Decay	α_{th}	α_{exp}	Fraction $(\Gamma_i/\Gamma)_{th}$	Fraction $(\Gamma_i/\Gamma)_{exp}$
$\Sigma^+ \to p\gamma$	-0.75	-0.76 ± 0.08	1.3	1.25 ± 0.07
$\Xi^- \to \Sigma^-\gamma$	0.57	—	0.14	0.127 ± 0.023
$\Lambda \to n\gamma$	-0.85	—	1.8	1.75 ± 0.15
$\Xi^0 \to \Lambda\gamma$	-0.23	0.4 ± 0.4	1.1	1.06 ± 0.16
$\Xi^0 \to \Sigma^0\gamma$	-0.03	0.2 ± 0.32	3.2	3.5 ± 0.4